Consider the piezoresisitive accelerometer shown in Fig. 1. The accelerometer housing is secured to a body undergoing the acceleration. In response to the acceleration of the body, the seismic mass accelerates causing the cantilever to bend. The bending applies strain to the piezoresisitors. Physical placement and orientation of the resistors is influenced by three considerations: the bridge configuration (which resistor is wired into which leg of the bridge), the region of maximum stress, and the direction of the strain to which we want the resistor respond.

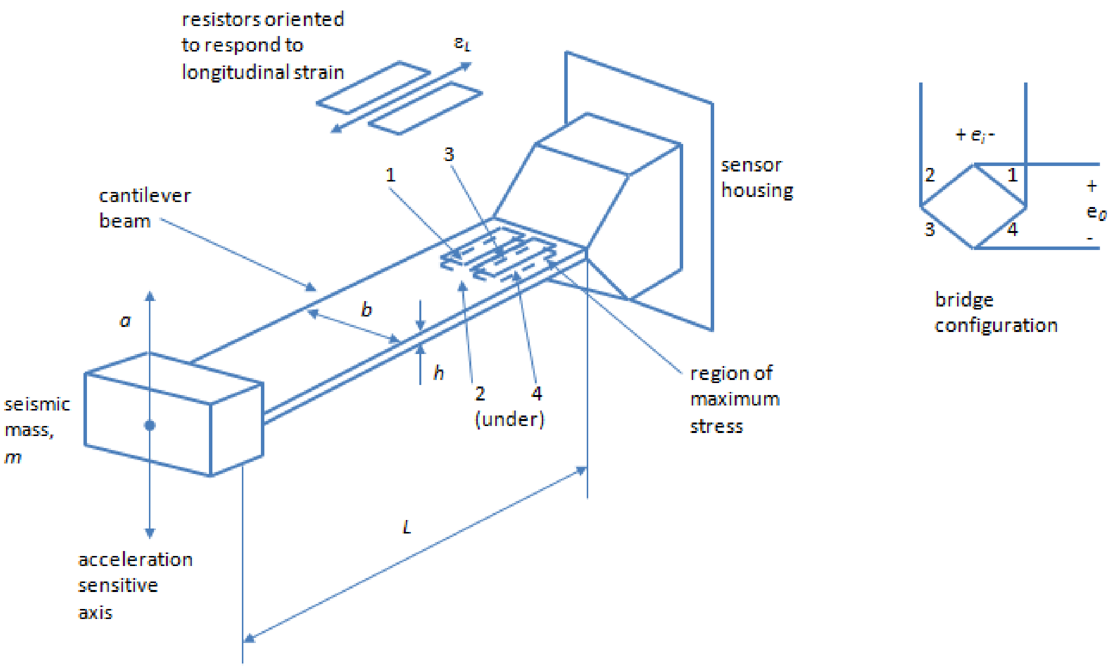

Fig. 1 Beam-type piezoresisitive single-axis accelerometer

The bridge configuration in this example is determined by the sign of the strain in each resistor location. If the beam bends downwards, the two resistors on top of the beam are in tension (a positive strain ε for p-type semiconductors) at the same time the resistors underneath the beam are in compression (a negative strain ε for p-type semiconductors). If the acceleration changes direction, the regions of tension and compression swap. For the strains to add up and not cancel each other out, we can place resistors 1 and 3 by side on one side of the beam (here both are shows on top of the beam) and resistors 2 and 4 on the other side of the beam. Then ε_1 and ε_3 have the same sign, opposite that of ε_2 and ε_4, and the bridge produces a total positive or a total negative output voltage Δe_0.

The four resistors are placed at the base of the beam because this is the region of maximum stress. Any other placement reduces the sensitivity of the sensor.

The resistors are oriented to be responsive to longitudinal strain, that is, the direction along the longitudinal axis of the cantilever beam. Transverse effects are negligible in this configuration.

A Wheatstone bridge is an electrical circuit that enables the detection of small changes in resistance. A voltage-sensitive, deflection-type circuit with a constant-voltage DC input, and ideal resistances (i.e., no impedance elements) in the arms of the bridge is considered. Such a bridge is shown in Fig. 2.

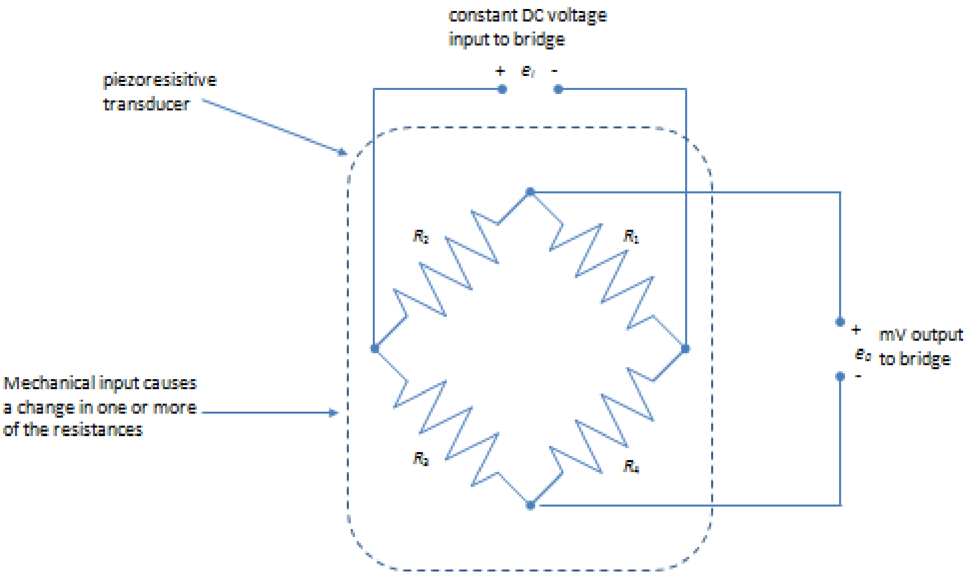

Fig. 2 Wheatstone bridge circuit inside a piezoresitive sensor

The sensor is designed such that the resistors undergo a small change in resistance ΔR that produces a small change in output voltage Δe_0. For the full-bridge with four identical piezoresitors, that is $R_1 = R_2 = R_3 = R_4 = R$, the relative output voltage Δe_0 with respect to the input voltage e_i is given by:

$$\frac{\Delta e_0}{e_i} \approx \frac{1}{4}\left(\frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R}\right). \tag{1.1}$$

In semiconductors, the relative change in resistance $\Delta R/R$ is due primarily to changes in resistivity $\Delta\rho/\rho$, not changes in geometry. For a piezoresistor subjected to longitudinal and transverse stresses, the resistivity change is:

$$\frac{\Delta\rho}{\rho} \approx \frac{\Delta R}{R} = \pi_l\sigma_l + \pi_t\sigma_t\,, \tag{1.2}$$

where σ_l and σ_t are the longitudinal and transverse stresses, and π_l and π_t are the longitudinal and transverse piezoresistance coefficients of the sensor material. In practice, longitudinal means "in the direction of current" and transverse means "perpendicular to the direction of current".

The longitudinal strains ε_l on the top and bottom of the beam shown in Fig. 1 are given by:

$$\varepsilon_l = \pm\frac{6maL}{Ebh^2}\,, \tag{1.3}$$

where dimensions *b*, *h* and *L* are shown in Fig.1; *m* denotes the seismic mass, *a* is the acceleration and *E* is the Young's modulus of the sensor material.

The stresses σ_l are given by the Hooke's law:

$$\sigma_l = E\varepsilon_l \, . \tag{1.4}$$

When selecting a particular sensor make and model, one usually has a choice of electrical output signals: 0-100 mV, 0-5 V or 4-20 mA and the maximum output voltage is denoted by $e_0^{\max}$. The DC excitation voltage is usually between 5-10 V and the maximum input voltage is denoted by $e_i^{\max}$. For a given maximum measured acceleration a_{max}, the sensitivity η of the bridge is given by:

$$\eta = \frac{\left(\frac{\Delta e_0}{e_i}\right) e_i^{\max}}{a_{\max}} \, . \tag{1.5}$$

To meet the goal of a full scale output $e_0^{\max}$, an amplifier circuit is added to attenuate (reduce) the bridge output by a factor:

$$K = \left(e_0^{\max} / \eta\right) . \tag{1.6}$$

This factor is called a gain *K* of the circuit.

The simplest op-amp circuit that provides positive attenuation comprises two inverting amplifiers in series, as shown in Fig. 3. A designer has to select the resistors R_f and R_0 to obtain the desired gain *K*.

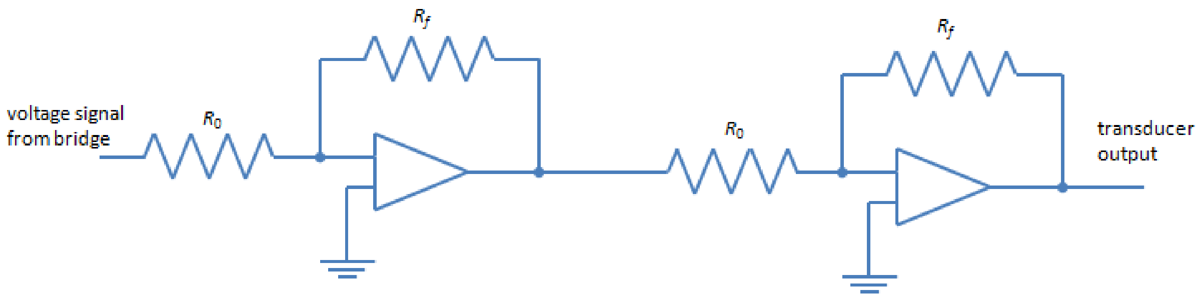

Fig. 3 Simple non-inverting attenuation circuit

The gain of an inverting amplifier is the ratio of R_f and R_0:

$$K_{amp} = -\frac{R_f}{R_0} , \tag{1.7}$$

and therefore the gain of the two inverting op-amps in series is given by:

$$K_c = \left(-\frac{R_f}{R_0}\right)\left(-\frac{R_f}{R_0}\right) = +\left(\frac{R_f}{R_0}\right)^2 . \tag{1.8}$$

The electrical output-to-input ratio with the amplifier gain is given by:

$$\left(\frac{\Delta e_0}{e_i}\right)_c \approx \frac{K_c}{4}\left(\frac{\Delta R_1}{R} - \frac{\Delta R_2}{R} + \frac{\Delta R_3}{R} - \frac{\Delta R_4}{R}\right) . \tag{1.9}$$

Readily-obtained resistors for use in an op-amp circuit (Ohms) are given below:

1.0	8.2	22	56	150	390
1.5	9.1	24	62	160	430
2.7	10	27	68	180	470
4.3	11	30	75	200	510
4.7	12	33	82	220	560
5.1	13	36	91	240	620
5.6	15	39	100	270	680
6.2	16	43	110	300	750
6.8	18	47	120	330	820
7.5	20	51	130	360	910

The percentage relative error of the required gain δ_K can be calculated as follows:

$$\delta_K = \frac{|K - K_c|}{K} \cdot 100\% \,. \tag{1.10}$$

δ_K does not exceed 2%.